Evaluating Methods of Symmetry

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Abstract

Skewness indicates a lack of symmetry in a distribution. Knowing the symmetry of the underlying data is important for parametric analysis, fitting distributions or doing transformations to the data. The coefficient of skewness is the commonly used measure to identify a lack of symmetry in the underlying data, although graphical procedures can also be effective. We evaluate five different methods to assess skewness: traditional coefficient of skewness index (S_1), skewness index based on the L-moments discussed by Hosking[1] (S_2, S_2'), the Quartile (S_4) and the Octile (S_4') skewness coefficients proposed by Hinkley[2], the asymptotic test of symmetry (S_3) developed by Randles *et al.*[3], and finally a novel intuitive approach based on the property of symmetric distributions (S_5). We have also developed a comprehensive and efficient SAS® macro for computing the various skewness measures and the appropriate power transformation, if one exists, to make an asymmetric distribution symmetric.[4]

1 Introduction

The first step in any statistical analysis includes summarizing the characteristics of the underlying data. All standard statistical packages routinely provide the summary statistics information, and this often includes a sample skewness score, which is a measure of symmetry. Symmetry is a rather complex property of probability distributions and it is difficult to identify deviations from it in a small number of observations. Broadly speaking, a dataset or a distribution is said to be symmetric if it looks same to the right and left of the center point. One of the numerous reasons for checking symmetry in a given data set is because many statistical tests rely strongly on the assumption of normality, which in turn relies on symmetry. Thus, a skewness measure can provide valuable information on issues such as data transformation, outlier detection, distribution fitting etc. so as to ensure that an appropriate analysis procedure (parametric versus non-parametric) is employed.

In this report, we present a novel approach to assessing symmetry, which is simple and computationally less complex in comparison to the asymptotic distribution-free test of symmetry developed by Randles *et al*.[3] This approach rests on the inherent characteristic of a symmetric distribution: the corresponding upper and lower percentiles are equidistant from its median (Section II). In section III, we introduce the other skewness measures namely the traditional coefficient of skewness index, skewness index based on the L-moments discussed by Hosking[1], the Quartile and the Octile skewness coefficients proposed by Hinkley[2], and the symmetry test developed by Randles *et al*.[3] In the past, several skewness indices have been explored individually and in this report we evaluate these competitors using extensive simulations (Section IV). We discuss the pros and cons of the different approaches and introduce our comprehensive SAS® macro (appendix I) that can perform these computations efficiently (Section V).

II. Proposed Measure of Symmetry

Hines and Hines[4] proposed a graphical power law transformation for transforming an asymmetric distribution symmetric based on the fundamental property of a symmetric distribution. Our method, described below, to assess the skewness in a given data set uses the same property of symmetric distributions.

Let $x_{(1)}, x_{(2)}, \ldots, x_{(n)}$ be the ordered random sample of size *n* from a distribution of the random variable X with mean μ and variance σ^2. As the first step, calculate the following:

x_M=median of the data set, i.e., 50th percentile

$x_{U_{100-i}}$ = upper percentiles of the data set (i = 0, 1, 2 …49)

x_{L_i} = lower percentiles of the data set (i = 0, 1, 2 …49)

Under symmetry, the corresponding upper and lower percentiles are equidistant from the median, i.e.,

$$x_M - x_{L_i} = x_{U_{100-i}} - x_M \Rightarrow x_M = \frac{x_{L_i} + x_{U_{100-i}}}{2}, \; i = 0,1,2,...,49$$

Thus, we obtain 50 estimates (Y_i) of the observed median, x_M, for any given data set. The number of percentiles can be chosen depending on the type of the data, and the sample size. Let m be the bound defined by $|1.96 \times \text{S.E.}(X)|$, where S.E.(X) is the standard error of the distribution of the original observations. Define:

$$p = \frac{\sum_{i=0}^{49} p_i}{50}, \text{ where } p_i = 1, \text{ if } |Y_i - x_M| \le m, \text{ and 0 otherwise, } i=0,1,2,...49$$

Thus, p is an estimate of the proportion of times the calculated median, Y_i, is close to the observed median, x_M, with a precision determined by m. Higher the value of p, the more symmetric is the underlying data. A reasonable ad hoc guideline for judging the symmetry in the underlying data set based on this approach is as follows:

- If $p \ge 0.9$, data is symmetric (i.e., only at most 10% of the original observations may be deviant)
- If $0.8 \le p < 0.9$, possibility of a few outliers in the data (i.e., 10% to 20% of the original observations may be deviant)
- $p < 0.8$, data is not symmetric and further investigation is needed (i.e., more than 20% of the original observations are deviant)

These cut offs can be fine tuned depending on how critical the assumption of symmetry is for the subsequent analyses. We evaluate the above cut offs by comparing them with the results from the symmetry test and other skewness measures section IV. One can also use the test of proportions to formally test and assess symmetry using our approach however that would require the choice of a null proportion for which the above guidelines can be used.

Previous work by Gastwirth[5] and Bhattacharya *et al.*[6] have looked at tests for symmetry based on modifications of the sign test and Wilcoxon tests. Our present approach is similar in principle to the above two methods in that it is also completely data driven, but is different in that it is based only on the property of a symmetric distribution.

In the remainder of this report, we will however focus only on some of the more commonly used skewness measures, the symmetry test and our proposed measure.

III. Skewness Indices

In this section, we will give a brief description of four different procedures used to compute skewness. We only present the formulae necessary to compute these indices / test statistics and the readers are referred to Hosking[1], Hinkley[7]and Randles *et al*.[2] for further details about the theoretical background.

The coefficient of skewness is defined as:

$$S_1 = \frac{m_3}{(m_2)^{3/2}}, \text{ where } m_r = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^r}{n}.$$

Here, m_r is the r^{th} sample moment about the sample mean. For symmetrical distributions, S_1 has expectation 0, i.e., when the data is symmetric, the sample skewness coefficient is near zero. If $S_1 > 0$, then the distribution is asymmetric with a positive skew and if $S_1 < 0$, then the distribution is asymmetric with a negative skew. The larger the absolute value of S_1, the more asymmetric is the distribution. (See Gupta[7] for a test based on this sample skewness coefficient).

The estimates of the sample L-skewness is given by the following:[1]

$$S_2 = \frac{l_3}{l_2}$$

where,

$$l_2 = 2w_2 - \bar{x},\ l_3 = 6w_3 - 6w_2 + \bar{x},\ w_2 = \frac{\sum_{i=2}^{n}(i-1)\, x_{(i)}}{n\,(n-1)}, w_3 = \frac{\sum_{i=3}^{n}(i-1)\,(i-2)\, x_{(i)}}{n\,(n-1)\,(n-2)}, -1 < S_2 < 1.$$

An alternative L-skewness index, $S_2' = \frac{1 + S_2}{1 - S_2}$, has also been defined by Hosking[1] and its properties have been discussed.[1,8] The index S_2' is easier to interpret than S_2, as it is the ratio of the length of the upper tail to the lower tail in samples of size 3. S_2' therefore ranges from 0 to ∞ and values of 1, > 1 and < 1 indicate symmetric, positively skewed and negatively skewed distributions. In the case of S_2, a value of 0 indicates symmetry, and $-1 < S_2 < 0$ indicates a negatively skewed distribution and $0 < S_2 < 1$ indicates a positively skewed distribution.

The Quartile (S_4) and the Octile (S_4') skewness coefficients proposed by Hinkley[2], given by:

$$S_4 = \frac{Q_{0.25} - 2Q_{0.5} + Q_{0.75}}{Q_{0.75} - Q_{0.25}}, \ 0 < S_4 < \infty$$

$$S_4' = \frac{Q_{0.125} - 2Q_{0.5} + Q_{0.875}}{Q_{0.875} - Q_{0.125}}, \ 0 < S_4' < \infty$$

For both (S_4) and (S_4'), values of 0, > 0 and < 0 indicate symmetric, positively skewed and negatively skewed distributions.

S_1, S_2 (S_2'), and S_4 (S_4') are measures of skewness (whose numerical values quantifies symmetry or asymmetry as the case may be), with no widely used test statistics associated with them. The distribution-free test of symmetry, however, tests if a univariate distribution is symmetric about some unknown value against a broad class of symmetric distribution alternatives.

To discuss the test statistic proposed by Randles *et al*.[3] based on the *n* unordered observations of X, first consider every triple (X_i, X_j, X_k), $1 \leq i < j < k \leq n$ (all the notations used in this paper are consistent with the discussion given in Wolfe and Hollander[9]). A set of three distinct observations is called a right triple when the middle observation is closer to the smallest than to the largest (and hence is skewed to the right) and is called a left triple when the middle observation is closer to the largest than to the smallest (and hence is skewed to the left). Define $f^*(X_i, X_j, X_k) = [\text{sign}(X_i + X_j - 2X_k)] + [\text{sign}(X_i + X_k - 2X_j)] + [\text{sign}(X_j + X_k - 2X_i)]$, where sign(y) = -1, 1, 0 if y is less than, greater than or equal to 0 respectively. If $f^*(X_i, X_j, X_k) = 1$, it is a right triple, it is a left triple if its value is -1 and it is neither a left nor a right triple if its value is 0. (Note that the test statistic is well-defined when zeros occur in the computation of ($X_i + X_j - 2X_k$; $\forall i, j, k$).) We then compute the following for the entire data set:

T = [number of right triples] - [number of left triples].

For each fixed t = 1…*n*, let

B_t = [number of right triples with X_t] - [number of left triples with X_t]

and for each fixed pair (s; t); $1 \le s < t \le n$, let

$B_{s,t}$ = [number of right triples with X_s, X_t] - [number of left triples with X_s, X_t].

The test statistic is based on the above combinations of the number of right and left triples in the entire data set and when *n* is large, its distribution is well approximated by the normal distribution. In particular, the test statistic S_3 is given by:

$$S_3 = \frac{T}{s}$$

where,

$$s^2 = \left[\frac{(n-3)(n-4)}{(n-1)(n-2)} \sum_{t=1}^{n} B_t^2 + \frac{(n-3)}{(n-4)} \sum_{s=1}^{n-1} \sum_{t=s+1}^{n} B_{s,t}^2 + \frac{n(n-1)(n-2)}{6} - \left\{ 1 - \frac{(n-3)(n-4)(n-5)}{n(n-1)(n-2)} \right\} T^2 \right].$$

Out of a sample of size *n*, there are ${}^{n}C_{2}$ distinct triples, and if the null hypothesis of symmetry holds, then we expect half of them to be right triples and half of them to be left triples. Roughly speaking, any substantial deviation in either direction (either more number of right triples or more number of left triples) is indicative of asymmetry in the underlying population. The null hypothesis of symmetry against the general alternative of asymmetry at a specified level of significance, α and large *n*, is rejected if $|S_3| \geq Z_{\alpha/2}$. Appropriate one-sided tests can be done to check for specific deviations (right or left skewness) from symmetry (see Wolfe and Hollander[9] for further details). Although computationally intensive, the results from this test are accurate even for small sample sizes and displays good power in detecting asymmetric distributions as compared to sample skewness measures.[3]

IV Simulations

We generated 1000 iterations of random samples of sizes 25, 50, and 100 from the following symmetric and positively skewed distributions: standard normal (mean=0, variance=1), and standard Cauchy (location=0, scale=1); Gamma (shape=8, rate=1), and Log Normal (mean=0.5, variance=1). The results are presented in Tables 1-3. There are variations in the results obtained from the summary skewness measures. A major issue with all the summary measures is quantification and the fact that there is a lack of consistency across the different skewness indices in terms of conclusions of asymmetry. We refer the readers to our previous work for other identified issues in these measures.[10]

We will focus on the comparison of the symmetry test with our proposed measure in the rest of this section. The symmetry test is a well defined test statistic that is accurate even for small sample sizes and displays good power in detecting asymmetric distributions as compared to sample skewness measures.[3] We will use a cutoff of 0.1 or higher to indicate symmetry, with values greater than 0.5 to indicate strong evidence of symmetry. Our proposed measure of symmetry also has guidelines for quantifying symmetry or asymmetry as the case may be (see section II).

In the case of the normal distribution, both the symmetry test and our proposed measure agree in that regardless of the sample size, the underlying distribution is symmetric. In the case of the Gamma distribution, although for N=25 and 50, the symmetry test p-values suggest symmetry (p=0.37, 0.25) based on a cutoff of 0.1, the values of 0.88 and 0.84 from our proposed measure indicates presence of possible asymmetry in the data as can be seen from Figure 1. In the case of N=100, both of them indicate asymmetry. In the case of the log normal distribution, both the symmetry test and our proposed method provide sufficient evidence for asymmetry in the distribution, although this is somewhat borderline for N=25 (see Table 1). The p-value from the symmetry test (0.12) and the estimated p from our measure (0.81) indicate possible asymmetry, which is expected as this data is generated from a positively skewed distribution. Figure 2 gives a representative plot of 5 samples generated from a log normal distribution of size N=25 with mean=0.5 and standard deviation=1. Clearly, even with such a small sample size, we see that the distribution is right tailed, i.e., positively skewed and both of the above approaches suggest possible asymmetry. However, based on Figures 1 and 2, we note that it is difficult to definitively conclude asymmetry or symmetry as the case may be, with such small sample sizes.

We discuss the case of the Cauchy distribution separately. The symmetry test suggests a strong evidence of symmetry (p-values ~0.5), whereas our proposed method gives values < 0.9 for N=25 and 50. To investigate this apparent disagreement further, we decided to tabulate the percentage of p-values ≥ 0.05, and ≥ 0.10 obtained from the 1000 iterations for the symmetry test and compare it with the estimated proportion from our approach (Table 4). Clearly, we can see that about 12% to 14% of the p-values from the symmetry test were in fact < 0.10, which is not captured by the average p-value. Although superficially the results from the two approaches appear contradictory, a more thorough analysis suggests that they are in fact consistent. It is therefore fair to conclude that the results from our proposed measure and the symmetry test in fact agree for the Cauchy distribution as well.

Hence, based on the simulation results, our proposed method is robust and provides a reasonable indication of symmetry or asymmetry even with small sample sizes. Moreover, the results from our approach are in agreement with the results from the symmetry test.

V Discussion

Accuracy and Interpretability The sample skewness coefficient and the Quartile and Octile skewness coefficients are sensitive to even small changes in the tail of the distribution, whereas the L-skewness, symmetry test and our approach are sensitive to changes in the shape of the main portion (in the middle as opposed to the tail). The sample skewness coefficient and the Quartile and Octile skewness coefficient is susceptible to moderate outliers in the sample since cubes of extreme deviations are highly influential. Royston[8] further demonstrated that the sample skewness coefficient is a poor estimator of skewness in skew distributions and suffers from several theoretical and practical disadvantages as compared to the L-skewness measure. The symmetry test is not effective at identifying asymmetry when sample sizes are small (< 20) [11], however using a Monte Carlo study, Randles *et al.*[2] showed that the symmetry test is superior to the test based on the sample skewness index. Our approach as well as the symmetry test and the L-skewness can provide a measure of relative skewness, whereas the other two skewness coefficients are less interpretable in terms of the distribution features.

Complexity The sample skewness coefficient is the easiest to compute, followed by the L-skewness, the Quartile/Octile coefficients, our approach and the symmetry test statistics. The L-skewness measure and our approach require the data to be sorted in an increasing order, whereas the symmetry test requires considering every triple of observations for computing the test statistic. When the sample size is large, although the symmetry test displays good power, it is computationally intensive compared to our approach.

Accessibility The sample skewness coefficient is part of many standard statistical packages and hence is accessible and also efficient in terms of time required for computation. Our readily available SAS® macro makes it feasible to compute all the other skewness measures discussed in this report, however there are tradeoffs between computational time versus power for the symmetry test, when the sample size is large.

The big question is: Which measure is appropriate? It probably suffices to say that this is situation dependent, particularly on how important the symmetry of the underlying data is for the purposes of the study. The first step would still be to do a quick plotting of the data to get a sense of its distribution. As we have shown, several factors like sample size, interpretability, complexity, and accessibility play a vital role in the selection of the skewness measure. Each measure has its own share of positives and negatives. The sample skewness coefficient, the L-skewness and the Quartile/Octile skewness measures are readily available (although not all of them are routinely produced as part of a statistical output, but can be coded easily and quickly) and computationally less intensive compared to the symmetry test. Between them, it has been shown that the L-skewness is more interpretable and less sensitive to extreme deviations in the tails. The symmetry test displays good power in detecting asymmetry against a broad class of symmetric distribution alternatives. Our novel approach to assessing symmetry is based on the fundamental property of a symmetric distribution. The test of proportions can be applied to our approach if a rigorous statistical hypothesis testing (and p-values) is required. A significant advantage of our method over the symmetry test is that it is intuitive, simple, and computationally easy, and yet provides results consistent as that of the comprehensive, computationally intensive symmetry test. If complexity and computational time are not constraints (mainly in case of large sample sizes), then the symmetry test is probably a good choice, however for a quick, practical and accurate assessment of symmetry, our proposed method is considerably better.

Our SAS® macro (see appendix I) provides a menu of options to perform all of the above computations efficiently and in addition also suggests an appropriate power transformation, if one exists, to make a data symmetric.[1] This latter part is noteworthy in that we not only address the problem i.e., identify whether a given data set has an underlying symmetric distribution, but also provide a solution i.e. an appropriate power transformation, if one exists, to make the asymmetric data symmetric.

Appendix I

Two slightly different versions of a macro are available for checking for symmetry.

1) %symmchk(ds = , chkvar =);

ds = name of the dataset
chkvar = variable to be checked for symmetry

The above macro computes the different symmetry measures discussed in this technical report and suggests the best power transformation, if one exists, to make an asymmetric distribution symmetric.

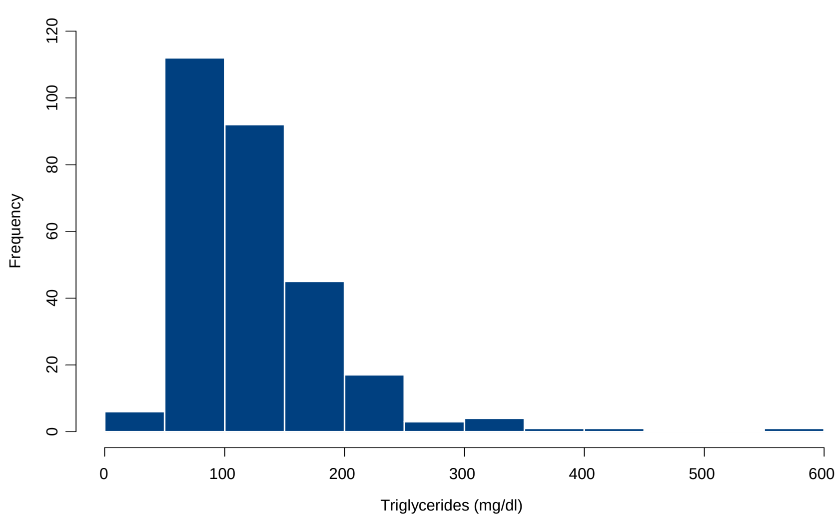
2) %symm(ds = , var =);

ds = name of the dataset
var = variable to be checked for symmetry

The above macro only computes the different symmetry measures discussed in this technical report for a given variable.

<u>Example</u>

Between January 1974 and May 1984, Mayo Clinic conducted a double-blinded randomized trial in primary biliary cirrhosis of the liver (PBC), comparing the drug D-penicillamine (DPCA) with a placebo.[12] We consider the variable, triglycerides (mg/dl), to illustrate the use of our macro. A histogram of this data (see below) reveals that this data is positively skewed.



The various skewness measures computed using the %symmchk() macro support our visual observation of asymmetry. See below for the macro call and the sample SAS output for this data.

%symmchk(ds = data, chkvar = trigly);

```
Var                    = TRIGLY
N                      =        282
S1: Sample Skewness    =  2.5104576
S2: L-Skewness I       =  0.2769045
S2':L-Skewness II      =  1.7658862
S3: Np Symmetry Test   =  9.1331897
S3:          P-value   =     0.0000
S4: Quartile Skewness=  0.2835821
S4':Octile Skewness    =  0.3333333
S5: New Method         =  0.4200000
```

Our macro then automatically performs several power transformations and checks for symmetry for the transformed data. The results are given in Table A. Looking at the p-values from the symmetry test and the new method, it appears that the log transformation makes the distribution of triglyceride variable symmetric (see table A for other suggested transformations). The histogram of the log transformed variable given below indicates a symmetric distribution as well.

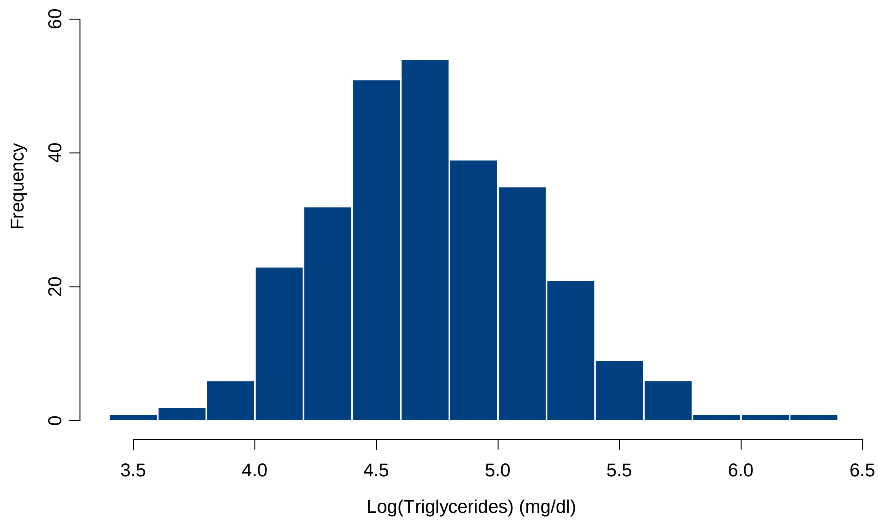


Table A: Results from the Different Power Transformations

Obs	Tranform EQ	S1 : Sample Skewness	S2 : L-Skewness I	S2': L-Skewness II	S3: NP symmetry Test	S3 P-value	S4: Quartile Skewness	S4: Octile Skewness	S5 : New Method
1	Original	2.5105	0.27690	1.7659	9.1332	0.00000	0.28358	0.33333	0.42
2	Square	6.8667	0.50375	3.0302	16.8925	0.00000	0.41467	0.54167	0.50
3	3rd power	11.4697	0.70467	5.7722	24.6288	0.00000	0.53196	0.70134	0.70
4	4th power	14.1747	0.85013	12.3445	32.0662	0.00000	0.63301	0.81322	0.84
5	square root	1.2176	0.16225	1.3873	5.3239	0.00000	0.21413	0.21462	0.46
6	3rd root	0.8931	0.12442	1.2842	4.0766	0.00005	0.19056	0.17359	0.52
7	4th root	0.7453	0.10561	1.2362	3.4613	0.00054	0.17871	0.15288	0.56
8	log	0.3582	0.05168	1.1090	1.6396	0.10110	0.14424	0.09245	0.86
9	1/square rt	0.3185	0.05618	1.1190	2.1163	0.03432	-0.07272	0.03274	0.82
10	1/3rd root	0.1030	0.02071	1.0423	0.8386	0.40170	-0.09663	-0.00896	0.98
11	1/4th root	-0.0075	0.00278	1.0056	0.2163	0.82877	-0.10856	-0.02985	0.98
12	1/original	0.9611	0.15926	1.3789	5.9872	0.00000	-0.00096	0.15596	0.66
13	1/square	2.5696	0.34762	2.0657	13.8746	0.00000	0.14039	0.38286	0.54
14	1/3rd power	5.1402	0.50854	3.0695	21.0387	0.00000	0.27443	0.56901	0.54
15	1/4th power	8.5350	0.64053	4.5637	27.3451	0.00000	0.39701	0.70920	0.60

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Table 1: Simulation Results (N=25, Iterations = 1000)

Distribution	S_1	S_2 (S_2')	Symmetry test (S_3) (stat / p-value)	S_4 (S_4')	Proposed Measure (prop. of times within the bound, m*)
Normal (mean=0, std=1)	0	0 (1.02)	0.007 (0.53)	0 (0)	0.93
Cauchy (location=0,scale=1)	-0.1	-0.02 (2.95)	-0.05 (0.5)	-0.01 (-0.01)	0.87
Gamma (shape=8, rate=1)	0.53	0.11 (1.27)	1.01 (0.37)	0.07 (0.11)	0.88
Log Normal (mean=0.5, std=1)	1.08	0.22 (1.61)	2.08 (0.12)	0.14 (0.24)	0.81

* m = $\left|1.96 \times \text{S.E.}(X)\right|$

Table 2: Simulation Results (N=50, Iterations = 1000)

Distribution	S_1	S_2 (S_2')	Symmetry test (S_3) (stat / p-value)	S_4 (S_4')	Proposed Measure (prop. of times within the bound, m*)
Normal (mean=0, std=1)	0	0 (1)	-0.008 (0.52)	0 (0)	0.93
Cauchy (location=0,scale=1)	-0.23	-0.03 (2.73)	-0.05 (0.47)	0 (-0.01)	0.89
Gamma (shape=8, rate=1)	0.61	1.11 (1.26)	1.57 (0.25)	0.07 (0.12)	0.84
Log Normal (mean=0.5, std=1)	1.34	0.24 (1.64)	3.35 (0.03)	0.17 (0.26)	0.72

* $m = \left|1.96 \times S.E.(X)\right|$

Table 3: Simulation Results (N=100, Iterations = 1000)

Distribution	S_1	S_2 (S_2')	Symmetry test (S_3) (stat / p-value)	S_4 (S_4')	Proposed Measure (prop. of times within the bound, m*)
Normal (mean=0, std=1)	0	0 (1)	-0.002 (0.51)	0 (0)	0.92
Cauchy (location=0,scale=1)	-0.18	-0.01 (2.28)	-0.02 (0.48)	-0.002 (0.002)	0.91
Gamma (shape=8, rate=1)	0.67	0.11 (1.27)	2.40 (0.09)	0.08 (0.13)	0.77
Log Normal (mean=0.5, std=1)	1.49	0.24 (1.64)	5.02 (0.001)	0.17 (0.28)	0.64

* m = $|1.96 \times S.E.(X)|$

Table 4: Comparison of the Simulation Results for the Cauchy distribution (location=0, scale=1)

Sample size	Symmetry test (percent p-values ≥ 0.05)	Symmetry test (percent p-values ≥ 0.10)	Symmetry test (stat / p-value)	Proposed Measure (prop. of times within the bound, m*)
25	91.6%	86%	-0.05 (0.5)	0.87
50	91.9%	85.8%	-0.02 (0.48)	0.89
100	93.1%	88.3%	-0.05 (0.47)	0.91

* m = $\left|1.96 \times \text{S.E.}(X)\right|$

Figure 1: Probability density plot of five replications generated from a gamma distribution (shape=8, rate=1) of size 25 (x axis = vector of points at which the density is estimated, y axis = density estimate at each x point)

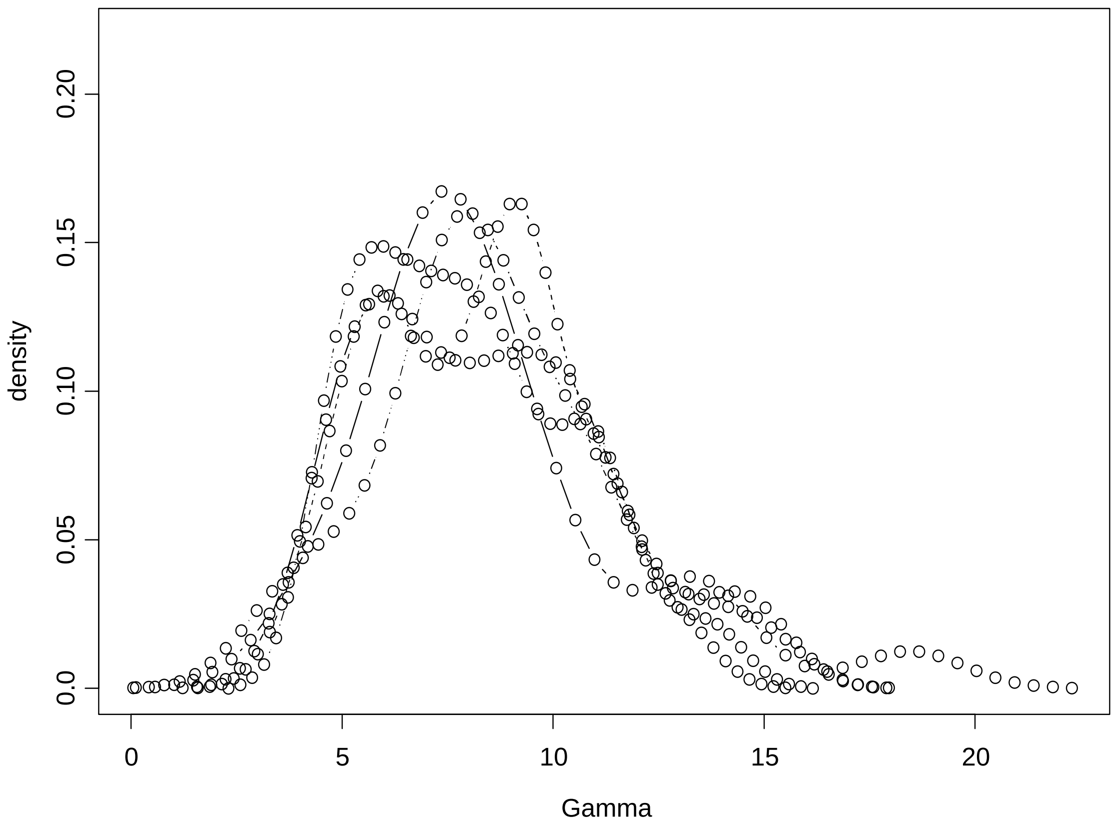

Figure 2: Probability density plot of five replications generated from a log normal distribution (mean=0.5, std=1) of size 25 (x axis = vector of points at which the density is estimated, y axis = density estimate at each x point)